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A NEW APPROACH TO UNIDIMENSIONAL POVERTY ANALYSIS: APPLICATION TO THE TUNISIAN CASE

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Fuzzy conceptualization of privation has been a step closer to more realistic handling of poverty. However, fuzzy approaches to poverty are still grounded on parametric axioms. Moreover, construction of poverty lines within these approaches still relies on ad-hoc methods. In this paper, we advance instead a fuzzy procedure based on the non-parametric bootstrap method, allowing us to depict fuzzy unidimensional privation states with boundaries drawn spontaneously from data. Fuzzy non-parametric measures of privation within each state as well as a collective fuzzy non-parametric index of poverty are derived, along with their corresponding confidence intervals. The new approach is applied to the analysis of poverty in Tunisia in 2005.

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1. INTRODUCTION

There is an international consensus on the necessity of poverty reduction strategies, aiming at empowering as many poor and marginalized people as possible. Indeed, the central objective of the Millennium goals agreed at the United Nations Millennium Summit in New York in 2010 is the halving of poverty by 2015. On the other hand, there is a wide recognition among researchers and policy makers that poverty is a complex notion. The tricky challenge is then to identify the "real poor" and devise realistic measures as faithful as possible to the intrinsic nature of poverty phenomenon, mainly that effective poverty monitoring relies on the accuracy of poverty measurement and analysis.

Traditionally, research on poverty has been based on a *clear* cut-off line, roughly dividing the population into "poor" and "non-poor." The choice of a poverty line is crucial for poverty measurement and analysis. The stochastic dominance approach to poverty has allowed us to circumvent uncertainty inherent to this choice by considering a range of poverty lines and a class of poverty indices instead of one precise index (see Foster and Shorrocks, 1988; Duclos *et al.*, 2008).

Under this classical framework, poverty has been viewed as an attribute characterizing an individual in terms of its presence or absence. A significant criticism has been made to this rigid poor/non-poor dichotomy. In recent years, a considerable literature on the fuzzy sets theory (Zadeh, 1965) has provided a new

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approach to allow for the matter of degree state of poverty and to manage the fuzziness and imprecision of poverty measurement.

Apart from this debate on the rough/smooth transition in the state of privation, another stream of research has tackled the controversy of unidimensional versus multidimensional conceptualization of poverty. The welfarist approach has been intensively used in the literature for the explanation and measurement of poverty. However, many authors have pointed out that it cannot alone fully capture individual living standards, both material and social. A considerable international research has adopted Sen's capability perspective (Sen, 1985) and has focused on multidimensional poverty by considering not only monetary variables but also other dimensions related to living conditions. We can cite, for instance, among others the works of Atkinson and Bourguignon (1982), Maasoumi (1986), Klasen (2000), and Bourguignon and Chakravarty (2003).

Fuzzy conceptualization of poverty has joined the unidimensional/multidimensional controversy. Several papers considering the application of fuzzy techniques to unidimensional poverty analysis have been published (see, for example, Shorrocks and Subramanian, 1994; Belhadj, 2011). Application of fuzzy techniques has also reached multidimensional approaches to poverty analysis (see Lemmi and Betti, 2006). Indeed, the works of Cerioli and Zani (1990), Cheli and Lemmi (1995), Naidoo *et al*. (2005), and recently Betti and Verma (2008) and Belhadj and Limam (2012) are considered among others as the major contributions in this field.

The scope of our work is unidimensional as we consider that most contributions to poverty analysis have been first conceived within the unidimensional framework to pave the way for potential extensions to the multidimensional perspective.

For both unidimensional and multidimensional approaches, identification of "the poor" remains a crucial step for the accuracy of poverty measurement and analysis. The arbitrariness of the value assigned to the poverty line has been pointed out in the literature as it has usually been assumed to be fixed and given in advance (Foster and Shorrocks, 1988). Furthermore, construction of poverty lines within fuzzy approaches to unidimensional poverty analysis has mainly relied on nutrition-based techniques, namely the Food Energy Intake Approach (Greer and Thorbecke, 1986a, 1986b) and the Cost of Basic Needs Method (Ravallion and Bidani, 1994). Many researchers have criticized the arbitrary foundation of these methods as calorie and basic needs depend on age and some other social demographic variables. Even so, the results brought out by both nutrition-based methods might be quite different as this has been pointed out by Wodon (1997). Indeed, the application of fuzzy unidimensional approaches to the measurement of poverty has been made within a parametric and axiomatic setting.

The purpose of our work is to fill the gap by advancing a novel approach that confers a non-parametric perspective to the application of fuzzy approaches to unidimensional poverty analysis at both levels of poverty identification and measurement.

The paper is organized as follows. The next section is devoted to the description of the first stage of this new approach, corresponding to the identification of the fuzzy non-parametric privation states. In Section 3, we move on to the second stage, namely the procedure used for the computation of fuzzy non-parametric unidimensional poverty, while in Section 4 we discuss the results of the application of this new approach to the Tunisian case. Section 5 concludes the study.

2. Fuzzy Non-Parametric Privation States

2.1. *Fuzzy Poverty*

Fuzzy set theory (Zadeh, 1965) has allowed us to move away from the poor/non-poor dichotomy and to consider instead the state of poverty or well-being of a person as a matter of degree. Cerioli and Zani (1990) have considered the group of poor households as a fuzzy set S, including the whole population with a membership function varying between 0 and 1, where if T denotes a universal set of objects—the range of all possible values for an input to a fuzzy system—then the membership function μ_S has the form: $\mu_S(t) : T \to [0, 1]$, where S is the fuzzy set and [0, 1] is the interval of real numbers from 0 to 1: $\mu_S(t) = 0$ if the element t does not belong at all to S whereas $\mu_S(t) = 1$ means that the element t completely belongs to S, and $0 < \mu_S(t) < 1$ if t partially belongs to S. The membership function allows to express this gradual membership. This makes its specification a systematic and basic step in the application of fuzzy set theory to poverty measurement. Indeed, Zadeh (1965) has defined a fuzzy set as "a class with a continuum of membership grades." The most commonly used forms for functional membership functions are piecewise linear, triangular, trapezoidal, and Gaussian shapes.

A crucial question in poverty research is the identification of the upper and lower bounds of the household well-being measure (i.e., income or expenditure). Absolute poverty thresholds based on nutrition-based methods (Greer and Thorbecke, 1986a, 1986b; Ravallion and Bidani, 1994) as well as relative poverty lines have been criticized of being ad-hoc and expert judgment dependent. Moreover, application of fuzzy approaches to unidimensional poverty measurement is still grounded on parametric and axiomatic modeling (see, for instance, Belhadj and Limam, 2012). Belhadj and Matoussi (2010) have proposed a parametric approach based on the configuration of three fuzzy privation states for the measurement of unidimensional poverty.

In this research, we propose to let the data suggest the appropriate poverty line instead of fixing it in advance. Consequently, the corresponding fuzzy privation states will be depicted from data and the procedure used for poverty measurement will be based on a data-driven approach instead of an axiomatic framework for the sake of more robust and reliable results. To this end, we combine fuzzy logic tools with non-parametric techniques to devise a fuzzy non-parametric procedure for the analysis and measurement of unidimensional poverty.

2.2. *Identification of Poverty States*

Step 1

We start by considering 100 fuzzy sets of poverty S_j for $j = 1, \ldots, 100$, where each one is defined as the set of couples $S = \{x, FN_S(x)\}$ for all $x \in$ the universe X, where X is the poverty predicate and where $FN_S(x)$ is the membership function associating a real number in the interval [0, 1] to each point of X. The value of the membership function represents the degree of belonging of x to S. The membership function should be non-increasing as an increase in welfare variables like income or expenditures translates to a reduction in the degree of poverty. Different methods have been used for constructing membership functions adapted to poverty measurement and analysis (see Cerioli and Zani, 1990; Chiappero-Martinetti, 2006). Indeed, the choice of the form of the membership function should convey the nature and the real features of the phenomenon. Here, we make the choice for a membership function form more adapted to the real features of poverty phenomenon (used in Belhadj and Matoussi, 2010) and which is defined as follows:

$$(1) \qquad FN(x, a, b, c) = \begin{cases} 0 & x < a, \\ 1 & a \le x < b, \\ x \cdot \dfrac{-1}{(c-b)} + \dfrac{(c)}{(c-b)} & b \le x < c, \\ 0 & x \ge c. \end{cases}$$

The parameters a, b, and c are set among estimators of the percentiles of the poverty predicate variable X, which are estimated via the bootstrap technique (Efron, 1979). Then, the membership functions corresponding to the 100 fuzzy sets are defined as follows:

For the first membership function defining the first fuzzy set:

$$FN_{S_1} = FN(x, 0, \hat{p}_1, \hat{p}_2),$$

where $\hat{p}_1$ and $\hat{p}_2$ are, respectively, the bootstrap estimates of the first and second percentiles of the poverty predicate variable which are set as equal to parameters b and c of the membership function specified in 1.

For membership functions FN_{S_2} to $FN_{S_{99}}$, where $j = 2, \ldots, 99$:

$$FN_{S_j} = FN\left(x, \hat{p}_{j-1}, \hat{p}_j, \hat{p}_{j+1}\right),$$

where $\hat{p}_{j-1}$, $\hat{p}_j$ and $\hat{p}_{j+1}$ are the bootstrap estimates of the $(j-1)$th, the (j)th, and the $(j+1)$th percentiles of the poverty predicate.

The last membership function is defined as follows:

$$FN_{S_{100}} = FN\left(x, \hat{p}_{99}, \frac{(\hat{p}_{99} + \hat{p}_{100})}{2}, \hat{p}_{100}\right),$$

where $\hat{p}_{99}$, and $\hat{p}_{100}$ are the bootstrap estimates of the 99th and the 100th percentiles of the poverty predicate. Hence, the membership grades of households $i = 1, \ldots, n$ to fuzzy states $S_1, \ldots, S_{100}$ are defined by the following membership grades matrix:

$$\begin{bmatrix} FN_{S_1}(x_1) & FN_{S_2}(x_1) & FN_{S_3}(x_1) & \cdots & FN_{S_{100}}(x_1) \\ FN_{S_1}(x_2) & FN_{S_2}(x_2) & FN_{S_3}(x_2) & \cdots & FN_{S_{100}}(x_2) \\ FN_{S_1}(x_3) & FN_{S_2}(x_3) & FN_{S_3}(x_3) & \cdots & FN_{S_{100}}(x_3) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ FN_{S_1}(x_n) & FN_{S_2}(x_n) & FN_{S_3}(x_n) & \cdots & FN_{S_{100}}(x_n) \end{bmatrix} \tag{2}$$

Step 2

The idea now is to detect a potential pattern arising from the fuzzy structure of data, shaped through the 100 poverty fuzzy sets. Initially, we propose to start by a visual sketch by plotting the values of the matrix of membership grades (2). Then, we go a step further by considering the analysis of this fuzzy structure via the scalar cardinality approach. The scalar cardinality of a fuzzy set is defined as the sum of the membership grades of the elements of the fuzzy set. This measure reflects the degree of containment of the elements in the fuzzy set. So, the higher the degree of containment of the elements in the fuzzy set, the larger the fuzzy cardinality measure and vice versa (Dhar, 2013). Hence, for each poverty fuzzy set S_j for $j = 1, \ldots, 100$ defined at Step 1, a scalar cardinality measure will be computed and this allows us to obtain consequently a sequence of cardinality measures.

Now, for investigating the existence of a potential categorization of the 100 fuzzy sets into k fuzzy states, we consider the application of the non-parametric approach of Matteson and James (2013) for the analysis of change points to the dataset of scalar cardinality measures. The approach does not make any assumptions about the distribution and the nature of the change beyond the existence of αth absolute moment for some α in (0, 2). The non-parametric estimation of both the number of change points in the sequence of scalar cardinality measures and the positions (corresponding to fuzzy sets) at which they occur is based on the divisive hierarchical algorithm which is included in the *ecp* package in the R statistical software. This algorithm focuses on detecting any distribution change within an independent sequence, based on a divergence measure (see section 2.1 in Matteson and James, 2013). Indeed, hierarchical significance testing is used to determine the statistical significance of change points and it is set as a stopping criterion for the iterative estimation procedure. An outline of this algorithm is provided in James and Matteson (2013).

3. Fuzzy Non-Parametric Unidimensional Poverty

Here, we go ahead with the presentation of the computation procedure for deriving the fuzzy non-parametric measures of privation within each state which leads us to the next stage of devising a collective fuzzy non-parametric index of poverty.

3.1. *Fuzzy Non-Parametric Privations Measures*

We describe below the procedure we follow to compute the fuzzy non-parametric measure of privation within each privation state.

Step 1

- Each household in the sample has a vector of 100 values corresponding to its membership grades to 100 membership functions (see Section 2). According to the results in Section 2, we decompose this vector into k vectors (corresponding to k fuzzy states) where each one includes membership grades corresponding to each fuzzy non-parametric privation state.

Step 2

- In order to derive the aggregate membership grade of the household over the fuzzy non-parametric privation state, we consider the union between fuzzy sets within the fuzzy non-parametric privation state and we apply therefore the maximum fuzzy operation to the vector including membership grades of households to the privation state, obtained at Step 1 (see Zadeh, 1965 for an overview of fuzzy sets operations).
- Compute a bootstrap estimate of the aggregate membership grade of the household over the fuzzy non-parametric privation state.

Step 3

- Repeat Steps 1 and 2 for all households of the sample.

Step 4

- In order to derive a fuzzy non-parametric measure of privation for each state, we take the weighted mean of the vector of aggregate membership grades of all households $i = 1, \ldots, n$ over the fuzzy non-parametric privation state. Let us consider A_{ij} as the aggregate membership grade of household i on the fuzzy non-parametric privation state j, where $j = 1, \ldots, k$. The fuzzy non-parametric measure of privation within each state is defined as follows:

$$Q_j = \frac{1}{N}\sum_i w_i A_{ij}.$$

  The weights w_i stand for the size of the household; namely, the number of people per household, and N stands for the total number of people in the sample.
- Then, we use the non-parametric bootstrap method to compute a bootstrap estimate of this statistic, and obtain as well the bootstrap percentile confidence interval of this measure, for each privation state.

The Bootstrap, the Jackknife, and other re-sampling techniques (Efron, 1982) have gained in popularity as a set of useful tools for applied statisticians and researchers all over the world. Indeed, the design complexity inherent to some surveys (stratification, unequal selection probabilities) have led to the development of design-based re-sampling methods such as the Taylor Linearization Method and the Jacknife Repeated Replication Method. These methods have been applied for variance estimation of complex indicators of poverty (see Osier, 2009; Verma and Betti, 2011). Given the difficulty with getting access to survey design information in most complex surveys, we opt for the bootstrap method as an alternative for variance estimation, in this research. The principle of the bootstrap method (Efron, 1979; Efron and Tibshirani, 1993) is to re-sample randomly with replacement from the data at hand to estimate the empirical distribution of statistics, without making any assumptions about their theoretical distributions. We have also used this method to derive bootstrap percentile confidence intervals for bootstrap estimates of the measures indicated above.

3.2. *The Collective Fuzzy Non-Parametric Index of Poverty*

The measurement of the total poverty is the sum of the k fuzzy states privation measures using the max–min convolution of fuzzy numbers. Let us assume for example that $k = 3$ corresponding to three fuzzy privation states that we choose to designate as a strong privation state, a medium privation state, and a weak privation state and let us consider Q_S as the fuzzy non-parametric measure of privation within the strong privation state, Q_M as the fuzzy non-parametric measure of privation within the medium privation state, and Q_W as the fuzzy non-parametric measure of privation within the weak privation state. We can then define the collective fuzzy non-parametric index of poverty as follows:

$$P = \bigvee (Q_S \wedge Q_M \wedge Q_W), \tag{3}$$

where ($\vee$), ($\wedge$) stand for the max–min convolution of fuzzy numbers. As indicated at Step 4 in Section 3.1, the procedure allows us also to derive bootstrap percentile confidence intervals for the estimates of fuzzy non-parametric privation measures within each state, for the level α, $\alpha \in [0, 1]$. We write bootstrap percentile confidence intervals of estimates of fuzzy non-parametric privation measures for the strong, medium, and weak privation states, respectively, as follows:

$$I_S = \left[L_{S\frac{\alpha}{2}}, U_{S\left(1-\frac{\alpha}{2}\right)}\right],$$

$$I_M = \left[L_{M\frac{\alpha}{2}}, U_{M\left(1-\frac{\alpha}{2}\right)}\right],$$

$$I_W = \left[L_{W\frac{\alpha}{2}}, U_{W\left(1-\frac{\alpha}{2}\right)}\right].$$

The bootstrap percentile confidence interval of the collective fuzzy non-parametric index of poverty P is defined by applying the min–max convolution on the lower and upper bounds of the confidence intervals defined above. The lower bound of the bootstrap percentile confidence interval of the collective fuzzy non-parametric index of poverty estimate is defined as follows for a level of α:

$$L_{P\frac{\alpha}{2}} = \bigvee \left(L_{S\frac{\alpha}{2}} \wedge L_{M\frac{\alpha}{2}} \wedge L_{W\frac{\alpha}{2}} \right).$$

As for its upper bound, it is defined as follows:

$$U_{P\left(1-\frac{\alpha}{2}\right)} = \bigvee \left(U_{S\left(1-\frac{\alpha}{2}\right)} \wedge U_{M\left(1-\frac{\alpha}{2}\right)} \wedge U_{W\left(1-\frac{\alpha}{2}\right)} \right).$$

Hence, the bootstrap percentile confidence interval for the bootstrap estimate of P is defined as follows:

$$I_P = \left[L_{P\frac{\alpha}{2}}, U_{P\left(1-\frac{\alpha}{2}\right)} \right].$$

4. Empirical Illustration

We consider a sample of the household budget data conducted by the Tunisian National Institute of Statistics (TNIS) in 2005, involving 12,317 households. The survey provides information about total annual expenditures in dinars per household as well as the size of each household. It also provides information about additional indicators of privation, namely the economic area as well as the social professional category of the household chief. Figure A.1 (see Appendix A) depicts the boxplot of the total annual expenditure variable. A brief summary of this variable is given in Table 1.

We define 100 fuzzy sets on the expenditures variable as indicated in Section 2. Then, we plot the membership grades matrix as shown in Figure B.1 (see Appendix B).

Figure B.2 (Appendix B) depicts the scalar cardinality measures corresponding to each fuzzy set. We use the divisive algorithm (see Section 2) to estimate the positions of the fuzzy sets at which occur the change points in this distribution.

This computation yields the fuzzy non-parametric boundaries of three fuzzy states as follows:

- Strong fuzzy non-parametric privation state: including fuzzy sets S_1 to S_{40}.
- Medium fuzzy non-parametric privation state: including fuzzy sets S_{41} to S_{70}.
- Weak fuzzy non-parametric privation state: including fuzzy sets S_{71} to S_{100}.

TABLE 1

Summary Statistics of the Total Annual Expenditure Variable

Minimum	First Quantile	Median	Mean	Third Quantile	Maximum
25	871	1,367	1,887	2,201	54,420

TABLE 2
STRONG, MEDIUM, AND WEAK FUZZY NON-PARAMETRIC PRIVATIONS (2005)

Privation State	$\hat{Q}$	$\left[L_{\frac{\alpha}{2}}, U_{\left(1-\frac{\alpha}{2}\right)}\right]$
Strong privation	0.352	[0.345, 0.359]
Medium privation	0.221	[0.215, 0.228]
Weak privation	0.186	[0.180, 0.192]

Note: $\hat{Q}$ stands for the estimate of the fuzzy non-parametric privation measure for each privation state, and $\left[L_{\frac{\alpha}{2}}, U_{\left(1-\frac{\alpha}{2}\right)}\right]$ stands for the bootstrap confidence interval of the corresponding measure for $\alpha = 0.05$.

Figures B.3, B.4, and B.5 (Appendix B) depict the aggregate membership grades of households with total annual expenditures lying within the boundaries of each fuzzy non-parametric privation state.

We move on afterwards to the computation of fuzzy non-parametric privations measures and the collective fuzzy non-parametric index of poverty. First, we examine results within the whole population. Then, we compare privations in rural versus urban areas and according to the social professional category of the household breadwinner.

According to the results in Table 2, the rate of unidimensional poverty in Tunisia in 2005 is estimated to be 22.1 percent. In fact, using the max–min convolution in (3) yields this estimate of the collective fuzzy non-parametric index of poverty:

$$\hat{P} = \bigvee(0.352 \wedge 0.221 \wedge 0.186) = (0.352 \wedge 0.221) \vee (0.352 \wedge 0.186) = 0.221.$$

Also, we are 95 percent confident that the true measure of the unidimensional poverty of Tunisian households in 2005 lies within the following bootstrap percentile confidence interval:

$$I_{\hat{P}} = [0.215, 0.228].$$

Indeed, histograms of bootstrap estimates of fuzzy non-parametric measures of privation within strong, medium, and weak privation states are shown, respectively, in Figures C.1, C.2, and C.3 in Appendix C.

As indicated above, the intensity of medium privation affecting Tunisian population in 2005 is estimated to be 22.1 percent. This intensity may denote for the Tunisian middle class, the high youth unemployment among graduates as well as the problems of corruption and illegal enrichment of the former regime's clan.

Indeed, the corruption among former president Ben Ali's clan as well as the high unemployment hitting young graduates has made the rich become richer and the poor get poorer. This has contributed to increasing the huge frustration among the population and led to the Tunisian revolution in 2010.

4.1. *The Regional Fuzzy Non-Parametric Poverty*

The estimated fuzzy non-parametric measures of privations for urban and rural areas are presented in Table 3. The results show that "strong privation" is more intense for households residing in rural areas. On the other hand, the membership grades of households residing in urban areas to "weak privation state" are more pronounced. Hence, any poverty alleviation policy in Tunisia should target as a priority poor households in rural and remote areas.

4.2. *The Fuzzy Non-Parametric Poverty by the Professional Social Category of the Household Chief*

Results based on Table 4 reveal that, on an average scale, agricultural workers and unemployed persons are the categories most affected by strong privation (strong privation intensity scores are about 55 and 60.5 percent, respectively).

TABLE 3

STRONG, MEDIUM, AND WEAK FUZZY NON-PARAMETRIC PRIVATIONS BY AREA (2005)

	$\hat{Q}_S$	$\left[L_{S\frac{\alpha}{2}}, U_{S\left(1-\frac{\alpha}{2}\right)}\right]$	$\hat{Q}_M$	$\left[L_{M\frac{\alpha}{2}}, U_{M\left(1-\frac{\alpha}{2}\right)}\right]$	$\hat{Q}_W$	$\left[L_{W\frac{\alpha}{2}}, U_{W\left(1-\frac{\alpha}{2}\right)}\right]$
Urban areas	0.251	[0.243, 0.259]	0.251	[0.241, 0.261]	0.258	[0.250, 0.268]
Rural areas	0.499	[0.488, 0.512]	0.178	[0.168, 0.190]	0.08	[0.073, 0.087]

Note: $\hat{Q}_S$, $\hat{Q}_M$, and $\hat{Q}_W$ stand respectively for estimates of fuzzy non-parametric privation measures for strong, medium, and weak privation states. The corresponding bootstrap confidence intervals of these estimates are defined for $\alpha = 0.05$.

TABLE 4

STRONG, MEDIUM, AND WEAK FUZZY NON-PARAMETRIC PRIVATIONS ACCORDING TO OCCUPATION (2005)

	$\hat{Q}_S$	$\left[L_{S\frac{\alpha}{2}}, U_{S\left(1-\frac{\alpha}{2}\right)}\right]$	$\hat{Q}_M$	$\left[L_{M\frac{\alpha}{2}}, U_{M\left(1-\frac{\alpha}{2}\right)}\right]$	$\hat{Q}_W$	$\left[L_{W\frac{\alpha}{2}}, U_{W\left(1-\frac{\alpha}{2}\right)}\right]$
High liberal profession	0.065	[0.046, 0.084]	0.161	[0.131, 0.190]	0.536	[0.508, 0.570]
Medium liberal profession	0.118	[0.090, 0.147]	0.221	[0.188, 0.258]	0.422	[0.386, 0.460]
Other employees	0.208	[0.231, 0.252]	0.294	[0.274, 0.318]	0.242	[0.221, 0.269]
Managers	0.257	[0.228, 0.284]	0.264	[0.240, 0.292]	0.241	[0.216, 0.266]
Self-employed persons	0.396	[0.360, 0.436]	0.235	[0.200, 0.270]	0.121	[0.098, 0.150]
Non-agricultural workers	0.460	[0.447, 0.473]	0.206	[0.194, 0.218]	0.092	[0.083, 0.102]
Farmers	0.468	[0.446, 0.490]	0.201	[0.180, 0.224]	0.092	[0.080, 0.105]
Agricultural workers	0.550	[0.511, 0.588]	0.143	[0.110, 0.181]	0.054	[0.034, 0.080]
Unemployed	0.605	[0.568, 0.645]	0.118	[0.083, 0.159]	0.034	[0.019, 0.054]
Pensioners	0.192	[0.172, 0.211]	0.276	[0.256, 0.298]	0.292	[0.273, 0.311]
Other inactive	0.347	[0.320, 0.369]	0.211	[0.189, 0.234]	0.197	[0.177, 0.219]
Support outside the household	0.388	[0.358, 0.416]	0.204	[0.177, 0.228]	0.163	[0.139, 0.190]

Note: $\hat{Q}_S$, $\hat{Q}_M$, and $\hat{Q}_W$ stand respectively for estimates of the fuzzy non-parametric privation measure for each state. The corresponding bootstrap confidence intervals of these estimates are defined for $\alpha = 0.05$.

On the other hand, these categories have the least privation scores on the weak privation state (weak privation intensity scores are about 5.4 and 3.4 percent respectively). Poverty policies in Tunisia should focus attention mainly on both categories as targeting them may yield a significant improvement of the welfare of the poor population.

5. Conclusion

In this paper, we have advanced a non-parametric view of the unidimensional fuzzy approach to poverty by combining the non-parametric bootstrap method with fuzzy logic tools. This has allowed us to circumvent the limits of the parametric setting of fuzzy approaches to unidimensional poverty analysis. Furthermore, depicting poverty thresholds from data has enabled us to get around the risk of underestimation/overestimation of poverty measurement. Indeed, the new methodology has brought out data-driven privation states, providing evidence for theoretical assumptions made in several works on the unidimensional fuzzy approach to poverty analysis.

We have applied this new approach to derive estimates of fuzzy non-parametric poverty measures for the case of Tunisian households in 2005. Then, we have gone a step further by providing an assessment of their accuracy by means of bootstrap percentile confidence intervals.

We consider the proposed methodology as a step toward a more realistic conceptualization of poverty. This may yield more accurate results for the sake of better targeted poverty alleviation strategies. Indeed, this approach is a valuable tool for researchers and policy makers to track the evolution of privations pattern within a society and to empirically assess the efficacy of poverty reduction strategies.

Potential extensions of this new procedure may include the multidimensional analysis of poverty within the framework of a more elaborated vision of poverty concept.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher’s web-site:

Appendix A: Total Annual Expenditure Variable
A.1: Boxplot of The Total Annual Expenditure Variable
Appendix B: Identification of Fuzzy Nonparametric Privations States
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